APPENDIX "A"

CERTIFICATE
OF
EXAMINATION
OF

FOUR 50 METRE STEEL SURVEYING TAPES.

For:- Commonwealth of Australia.

Sent by:- Messrs. James Chesterman & Co. Ltd.

Marked:- No.1. № 60182, No.2. № 60183,
No.3. № 60184, No.4 № 60185.

Each tape is also marked:-

CHESTERMAN, SHEFFIELD, ENGLAND.

Description:- The four steel tapes are of section 1/8" × 1/50" and are fitted at their ends with terminals which serve for connecting them to an electrical resistance bridge. The tapes are used in pairs, and the bridge is provided for measuring the resistance of a pair of tapes so as to enable corrections for changes in their mean length due to changes of temperature to be made on the basis of the corresponding changes in their joint resistance. In service the two tapes of a pair are connected together electrically at one end by a short flexible lead of resistance 0.005 ohm and at the other end each tape is connected to the bridge by a flexible lead of resistance 0.1 ohm. The bridge is used to measure the combined series resistance of the two tapes plus the short flexible, allowance for the other two flexibles being incorporated in two of the bridge coils.

On each tape short scales are engraved at 0, at 66 feet, and at 50 metres; each scale is 0.4 foot long

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Director.

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and is sub-divided to 0.002 ft., the main sub-division lines being figured 0, 1, 2 ..... 20.

The conditions of support of the tapes are as follows:-

(1) When using the 0 - 50 m intervals.

The two tapes (of either pair) are suspended in double catenary under a tension of 20 lb. 10 oz. applied tangentially at the ends of each tape; throughout their lengths the two tapes are kept as close together as possible with the central support for each tape in the same horizontal plane as the terminal scales.

(2) When using the 0 - 66 feet intervals.

The tape is supported at 66 feet on a roller and a tangential tension of 20 lb. 10 oz. applied to its ends. The roller should be in the same horizontal plane as the terminal scale at the zero end.

The resistance bridge, No.32978 № 60186, has been tested at the Laboratory and the results of the test are given in a separate Certificate of Examination dated 30th December 1938, Ref: E.T.D. 197/14.

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These tapes have been tested at The National Physical Laboratory, in pairs, and under the conditions of support set out above, with the object of determining for each pair:-

(1) the mean length of the intervals 0 - 50 m on the two tapes for a particular value of their combined resistance

(2) the relationship between change of length and change of resistance when the temperature of the pair changes.

A description of the procedure used in the tests is given in the Addendum to this Certificate and is followed by a discussion of the results obtained. Reasons are given for adopting in combination Tapes 1 and 3 to form one pair and Tapes

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2 and 4 to form the second pair. The final results are given below in the form of an equation expressing length in terms of resistance. In these equations l is the mean length of the two tapes measured between corresponding lines on the scales at 0 and 50 metres, and r is the combined resistance of the two tapes plus the short flexible, as determined by means of the bridge No. 32978 & 60186, the bridge being at a temperature of 68°F. approximately.

Tapes 1 and 3.

$$l = 49.994\,620 + 0.013\,64\,(r - 13) \text{ metres}$$
$$= 164.024\,63 + 0.044\,75\,(r - 13) \text{ feet}$$

Tapes 2 and 4.

$$l = 50.001\,415 + 0.014\,09\,(r - 13) \text{ metres}$$
$$= 164.046\,92 + 0.046\,23\,(r - 13) \text{ feet.}$$

The values of l computed by means of these equations may be regarded as being accurate to within ±1 part in a million for values of r ranging between 13.2 and 14.4 ohms for Tapes 1 and 3; between 12.7 and 13.9 ohms for Tapes 2 and 4. Measurements have not been made for lower values of r but it is considered that the equations can be used for lower values without much loss of accuracy.

The length of the interval 0 to 66 feet, under the conditions of support specified on page 2, has been determined for each tape. The observations were distributed so as to obtain measurements for a number of scale combinations (0 to 0, 5 to 5, 10 to 10 etc.) and the mean length found for each tape is given in the following table. The measurements were made at a temperature of about 65°F. and corrected to 68°F. by using the mean coefficient of linear thermal expansion found for the two sample lengths (see below). The

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values given in the table may be regarded as accurate to within ±2 parts in a million.

Tape No.	Length of Interval 0 - 66 feet at 68°F.
	feet.
1	66.000 $_{20}$
2	65.999 $_{76}$
3	66.000 $_{54}$
4	65.999 $_{90}$

The coefficients of linear thermal expansion of two samples of the tapes, each about 30" long, have been determined. These samples were stated by the manufacturers to have been cut from the same rolling as the four 50 m tapes, one prior to and the other subsequent to the cutting from the roll of the four tapes. Measurements were made over the temperature range 33°F. to 91°F. approximately, each sample being kept under a constant tension of about 20 lb. 10 oz. The two samples gave identical results, within the limits of experimental error, and the value of the coefficient of each was found to be

0.000 006 1_2 per 1°F.

The scales engraved on the tapes have been examined and have been found to be satisfactorily graduated. The measurements made showed **that**:-

(1) the errors **of** sub-division of the scales are small and may be regarded as negligible

(2) the actual overall length of each scale, under a tension of 20 lb. 10 oz. is slightly shorter than the nominal length (0.4 foot), the average value found for the overall length being 0.3999 ft.

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N.B. An error in the overall length of the scale gives rise to an error in the measurements made with the tape only when the lengths measured differ appreciably from the nominal length (50 m or 66 ft.) e.g. if the measured length exceeds the nominal by 0.4 ft. the result obtained will be too big by 0.0001 ft. on account of scale error.

The following additional information has been obtained in the course of the tests on the tapes:-

(1) The mass per unit length of each of the tapes was determined from weighings of the tapes made before the electrical terminals were fitted to them. The results obtained were:-

Tape No.	Mass of 1 metre.	Mass of 1 foot.
	(lb.)	(lb.)
1	0.028 27	$0.008\ 61_7$
2	0.028 38	$0.008\ 65_1$
3	0.027 07	$0.008\ 25_0$
4	0.028 73	$0.008\ 75_6$

(2) The change in length of the 0 to 50 m interval produced by small variations in tension from the value 20 lb. 10 oz. weight has been found, for each tape, to be:-

+0.000 008 8 metre or +0.000 028 9 foot for an increase in tension of 0.01 lb. weight.

For a pair of tapes, when measurements are referred to their mean length, allowance for a change of tension should be made at the same rate provided both tapes remain subject to equal tension.

For the interval 0 to 66 ft. on each tape, the correction for change of length due to change of tension can be computed on a proportional basis viz. 66 : 164.

(3) The straining cords and swivels supplied for use with the tapes have been weighed. The mass of the cord has been found to be:-

0.011 lb. per metre or 0.0033 lb. per foot.

N.B. A tangential tension of 20 lb. 10 oz. weight has been assumed for the purpose of standardising the tapes. If the mass of each of the weights used for tensioning the tapes is 20 lb. 10 oz., allowance for the increase in tension

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introduced by the cord on the weight side of the pulley should be made on the basis of the information given in (2) and (3).

NOTES.

(1) The lengths of the 0 to 50 metre intervals on the tapes have been determined by reference to the Laboratory's Metric Standards of Length. For the purpose of converting from metres to feet, the relationship accepted by the Laboratory, viz:-

1 metre = 3.280 845 6 feet.,

has been used.

(2) The lengths of the tapes have been determined under a gravity value g = 981.19 cm per sec. per sec. When the tapes are used at a base where g has a different value, it is necessary to introduce a correction.

This correction should be applied at the rate of

0.000 014 0 metre or 0.000 046 foot

for a 1 cm per sec. per sec. change in g, when measurements are referred to the mean length of the 0 to 50 m intervals on a pair of tapes, the correction being positive if the value of g at the base is greater than 981.19 cm per sec. per sec.

For the 0 to 66 ft. intervals a proportional correction, in the ratio 66: 164, should be applied.

(3) When the tapes are used in double catenary under the conditions of support specified on p.2, the terminal scales (at 0 and 50 m) are inclined to the horizontal at an angle of 1° 0'. If, therefore, the tapes are used to measure lengths differing appreciably from the nominal length (50 m), correction should be made for the cosine error introduced on account of the inclination of the scales.

(4) In determining the lengths of the tapes, only the ends of the graduation lines at or near the edge of the tape have been viewed.

(5) When using the tapes, the graduation lines on which readings are taken should be horizontal.

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ADDENDUM.

An account of the experimental work carried out in the tests on the four tapes is given below and is followed by an examination of the results obtained and the conclusions drawn from them.

Notation used.

Let l = length of any one tape.
r = resistance of this tape.
Then $\frac{\partial l}{\partial r}$ = its rate of change of length with resistance.

If two tapes of lengths l_A and l_B, resistances r_A and r_B, are used in combination, and measurements of l_A and l_B are associated with measurements of the combined resistance $r_A + r_B$, the rates of change of length with resistance obtained from the measurements will be:-

$$\frac{\partial l_A}{\partial(r_A+r_B)} \text{ for tape A,} \quad \frac{\partial l_B}{\partial(r_A+r_B)} \text{ for tape B.}$$

Determination of Relationship between Change of Length and Change of Resistance.

The experiments first made were directed towards determining $\frac{\partial l_A}{\partial(r_A+r_B)}$ and $\frac{\partial l_B}{\partial(r_A+r_B)}$ for tapes 1 and 2 used as one pair and tapes 3 and 4 used as the second pair.

For this purpose, each pair of tapes was set up, in turn, on the Laboratory's 50 metre tape bench, the conditions of support being as specified on p.2. The temperature of the tapes was changed by passing an electrical current through them, and simultaneous values of length and resistance were recorded. Observations were taken for steady currents of 0.2 amp., 3.5 amp. and 5 amp.

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respectively, the initial current producing only a very small heating effect but being necessary for operating the bridge when determining the resistance of the tapes at normal temperature. The lengths of the 0 to 50 m intervals on the tapes under these conditions were determined and the combined (series) resistance of the tapes measured simultaneously on the bridge.

By these means, the temperature of the tapes was changed over the range 60°F. to 112°F. approximately, corresponding to changes of length and resistance of about 16 mm and 0.6 ohm respectively for each tape. The results obtained showed that $\frac{\partial l_1}{\partial(r_1+r_2)}$ and $\frac{\partial l_2}{\partial(r_1+r_2)}$ were identical, within the limits of experimental error, but that $\frac{\partial l_3}{\partial(r_3+r_4)}$ exceeded $\frac{\partial l_4}{\partial(r_3+r_4)}$ by about 10%.

In order to investigate this more fully, the electrical apparatus was modified so as to enable the individual as well as the combined resistances of the tapes to be determined and the measurements were repeated over a similar range of temperature. From the results obtained, the resistance of each tape at 18.5°C. (65.3°F.) and its rate of change of length with resistance $\frac{\partial l}{\partial r}$ were computed; the values found are given in the following table.

Tape No.	Resistance at 18.5°C.	$\frac{\partial l}{\partial r}$
	ohms.	mm per ohm
1	6.447	27.83
2	6.465	28.01
3	6.866	26.76
4	6.348	28.34

For each tape the plot of l against r was found to be a straight line over the range of resistance investigated but the value of

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the slope $\frac{\partial l}{\partial r}$ and of the resistance at 18.5°C. varied from tape to tape as shown in the table.

On account of their difference in resistance, the two tapes of a pair, when heated electrically by a current passed through them in series, were not heated equally and for this reason it was not possible to deduce from the results of the experiments the true values of $\frac{\partial l_A}{\partial(r_A+r_B)}$ and $\frac{\partial l_B}{\partial(r_A+r_B)}$ when the tapes are subject to a common change of temperature. It was possible, however, to compute accurately the required value of $\frac{\partial(l_A + l_B)}{\partial(r_A + r_B)}$ for each pair of tapes, the theoretical basis of this computation being as follows:-

Let $\frac{\partial l}{\partial r}$ for tape A $= k_A$

and $\frac{\partial l}{\partial r}$ for tape B $= k_B$

Then $$l_A = k_A r_A + C_A$$

in which C_A is a constant.

If L_A and R_A are corresponding values of length and resistance for tape A,

$$L_A = k_A R_A + C_A$$

and, therefore,

$$l_A = L_A + k_A(r_A - R_A)$$

Similarly, $$l_B = L_B + k_B(r_B - R_B)$$

and hence,

$$l_A + l_B = L_A + L_B + (k_A r_A + k_B r_B) - (k_A R_A + k_B R_B)$$

Assume $$r_B = m r_A$$

and $$k_B = n k_A$$

Then $$k_A r_A + k_B r_B = (1 + mn) k_A r_A$$

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Assume also that

$$k_A r_A + k_B r_B = K(r_A + r_B)$$

i.e. $$k_A r_A + k_B r_B = K(1 + m) r_A$$

Then $$K = \frac{1 + mn}{1 + m} k_A$$

In this expression for K, k_A and n are constant and have been determined in the experiments described above; m is a variable quantity (because the tapes have different coefficients of increase of resistivity with temperature), but since n is approximately unity even a large change (e.g. 10%) in m would lead to an inappreciable change in K. As the actual changes found in m were of the order 0.1%, it is clear that the experimental data collected enables K to be accurately determined and the equation for $l_A + l_B$ can be expressed, without loss of accuracy, in the form:-

$$l_A + l_B = L_A + L_B + K\,(r_A + r_B) - (R_A + R_B)\;.$$

As is shown in the table above, tape No.3 has a distinctly smaller value of $\frac{\partial l}{\partial r}$ than the other three tapes. In combining the tapes into pairs, tape No.3 has, therefore, been associated with tape No.1, which has the next lowest value of $\frac{\partial l}{\partial r}$, and the values of K have been computed for Tapes 1 and 3 used as one pair, and for Tapes 2 and 4 used as the second pair.

Standardisation of the 0 - 50 m intervals.

The standardisation of the 0 to 50 m intervals on the tapes was carried out at a temperature of about 65°F. i.e. for values of $r_A + r_B$ equal to 13.31 ohms approximately for Tapes 1 and 3 and

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12.81 ohms approximately for Tapes 2 and 4. Measurements of the resistance of each pair of tapes were made by means of the resistance bridge No.32978 № 60186; the length measurements were made by reference to the Laboratory standards, the observations being distributed so as to obtain a mean value corresponding to a number of different scale combinations (e.g. 0 to 0, 5 to 5, 10 to 10 etc.). In order to simplify the computation of corrections when using the tapes in the field, the results of the standardisation have been calculated for a value of $r_A + r_B$ equal to 13 ohms exactly (viz. $R_A + R_B$ = 13 ohms) and have been given (on p. 3) in the form of an equation expressing the mean length l of a pair of tapes in terms of their joint resistance r.

C. G. DARWIN,

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